D Thermistor response

To derive the response of the thermistor to changes in the bulk temperature of the fluid, the following situation is considered:

$$
\begin{aligned}
T &= T_0 && \text{for} \quad t < -\frac{t_{heat}}{2} \\
T &= T_0 + a\left(t + \frac{t_{heat}}{2}\right) && \text{for} \quad -\frac{t_{heat}}{2} < t < \frac{t_{heat}}{2} \\
T &= T_0 + a t_{heat} && \text{for} \quad t > \frac{t_{heat}}{2}
\end{aligned}
\tag{D.1}
$$

This corresponds to a constant heat input between $t = -t_{heat}$ and $t = t_{heat}$ with an isentropic response in the bulk fluid. By taking the Fourier transform, the spectrum density $S(\omega)$ at angular frequency ω is obtained, which is:

$$S(\omega) = -\frac{ia}{\pi\omega^2}\sin\left(\omega\frac{t_{heat}}{2}\right) + \left(a\frac{t_{heat}}{2}\right)\delta(0) \tag{D.2}$$

where $\delta(x)$ is the Dirac Delta function. Fourier analysis is employed here because the solution of the heat flow problem in which the bulk temperature has a simple harmonic variation with time is known, as a standard problem in linear acoustics. Equation (D.2) shows that the Fourier spectrum is heavily weighted towards low frequencies (much as expected). If we assume now that the thermistor is a homogeneous sphere of radius R then the temperature is:

$$T = T_0 + A\mathrm{j}_0(k_{th}r)\exp(i\omega t) \tag{D.3}$$

with j_0 is the spherical Bessel function of order zero, $k_{th} = \sqrt{-i\omega/D_{th}}$, D_{th} is the thermal diffusivity of the thermistor material, r is the radial coordinate and A is a constant. For $r > R$ it is then:

$$T = T_0 + T_1\exp(i\omega t) + \frac{B}{r}\exp\{i(\omega t - k'r)\} \tag{D.4}$$

where the first term is the initial temperature, the second is the Fourier component in the bulk, and the third is the thermal boundary layer at the surface of the thermistor (B is a constant). The prime denotes properties of the fluid. Note that the thermal boundary layer takes the form of an outward-going thermal wave which attenuates rapidly with increasing r because of the imaginary part of k'.

The two boundary conditions which determine the constants A and B are those requiring continuity of temperature on and of heat flow through the surface $r = R$. The result is that:

$$Aj_0(kR) = \frac{T_1}{1+f} \tag{D.5}$$

where

$$f = -\left(\frac{\lambda}{\lambda'}\right)\left(\frac{kR}{1+ik'R}\right)\frac{j_1(kR)}{j_0(kR)} \tag{D.6}$$

Notice that, if $f \ll 1$, the surface temperature of the thermistor follows almost exactly the bulk temperature of the fluid at angular frequency ω. Now, as $T \to T_c$, the ratio λ/λ' goes to zero and, since $D' \equiv D_T \to 0$ the term $kR/(1+ik'R)$ also vanishes. Furthermore, as $\omega \to 0$, both the term $kR/(1+ik'R)$ and the ratio of Bessel functions go to zero.

The net result of all this calculation is that, the surface temperature of the thermistor should follow that of the bulk fluid more and more closely as $T \to T_c$.