B The PE near the heater

This appendix is devoted to the calculation of the relative contribution of the PE to the temperature field near the heater for the geometry of our set up. We set out to determine the significance of the shaded area in fig. 2.3. The ratio between this shaded area and the temperature field in the isobaric case is offered by eq. (2.42). We start off with the term that is given by the r.h.s. of eq. (2.42):

$$\frac{\dfrac{\sigma_h S_h}{1+\sigma_h}}{\displaystyle\sum_{i=1}^{N} \frac{\sigma_i S_i}{1+\sigma_i}} \cdot \frac{G(x_h^*, t^*)}{2\sqrt{t^*}\ \mathrm{ierfc}\left(\dfrac{x_h^*}{\sqrt{t^*}}\right)}, \tag{B.1}$$

where the subscript i refers to a wall segment i, the subscript h refers to the heater substrate $(i = h)$, σ_i denotes the inverse thermal impedance ratio between fluid and the ith wall material, S_i denotes the surface area of the ith wall material, G is the function given by eq. (A.16), x_h^* is the reduced space coordinate at the heater (eq. (2.40)) and t^* the reduced time coordinate (eq. (2.28)). The term (B.1) is a product of two ratios which are treated separately.

The second ratio in (B.1) is a ratio between two smooth functions which may be rewritten to:

$$R\,(x_h^*, t^*) \equiv 1 - \frac{\mathrm{erfc}\left(\dfrac{x_h^*}{\sqrt{t^*}}\right) - \exp(2x_h^* + t^*)\ \mathrm{erfc}\left(\dfrac{x_h^*}{\sqrt{t^*}} + \sqrt{t^*}\right)}{2\sqrt{t^*}\ \mathrm{ierfc}\left(\dfrac{x_h^*}{\sqrt{t^*}}\right)}. \tag{B.2}$$

With $y \equiv x_h^*/\sqrt{t^*}$ and $\mathrm{ierfc}(x) = \exp(-x^2)/\sqrt{\pi} - x\,\mathrm{erfc}(x)$, (B.2) may be rewritten to:

$$R = 1 - \frac{\left[1 - \frac{\exp(y^2 + \sqrt{t^*})\operatorname{erfc}(y + \sqrt{t^*})}{\exp(y^2)\operatorname{erfc}(y)}\right] / \sqrt{t^*}}{\frac{1}{\frac{\sqrt{\pi}}{2}\exp(y^2)\operatorname{erfc}(y)} - 2y} . \tag{B.3}$$

In 'Handbook of mathematical functions' [102] the following inequality is presented:

$$\frac{1}{x + \sqrt{x^2 + 2}} < \frac{\sqrt{\pi}}{2}\exp(x^2)\operatorname{erfc}(x) \le \frac{1}{x + \sqrt{x^2 + \frac{4}{\pi}}} \qquad (x \ge 0) . \tag{B.4}$$

Utilizing this inequality, we may rewrite (B.3) to

$$R = 1 - \frac{\sqrt{t^*} + \sqrt{(y + \sqrt{t^*})^2 + a_2} - \sqrt{y^2 + a_1}}{\sqrt{t^*}(y + \sqrt{t^*} + \sqrt{(y + \sqrt{t^*})^2 + a_2})(\sqrt{y^2 + a_1} - y)} , \tag{B.5}$$

with $4/\pi \le a_i < 2$ and $a_2 \ge a_1$.

When $x = 0$, $y = 0$ in which case $a_1 = 4/\pi$ and (B.5) reads:

$$R(0, t^*) = 1 - \frac{1}{\sqrt{t^*}}\left[\frac{1}{\sqrt{a_1}} - \frac{1}{\sqrt{a_2 + t^*} + \sqrt{t^*}}\right] . \tag{B.6}$$

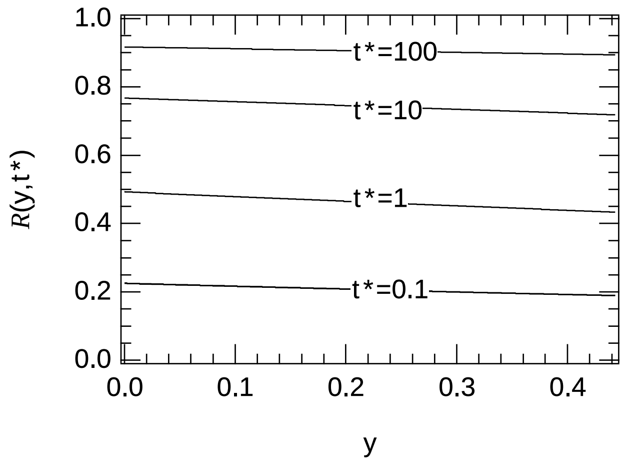
Since $a_2 \ge a_1$ this term is always smaller than 1. For $t \to \infty$, immediately we see that this term asymptotically goes to 1. A minimum is found close to $t^* = 1$ where $R(0, 1) \cong 0.5$. For $t \to 0$, $a_2 \to a_1$ and $R(0, t \to 0) \to 1$.

When $x \to \infty$ or $t \to 0$, $y \to \infty$ in which case $a_1 \to 2$ and, consequently, $a_2 \to 2$. We see that for $\sqrt{t^*} << y \to \infty$ and $a_1 \cong a_2 \to 2$ (B.5) reduces to

$$R = 1 - \frac{1}{(y + \sqrt{y^2 + 2})(\sqrt{y^2 + 2} - y)} = 1 - \frac{1}{y^2 + 2 - y^2} = \frac{1}{2} . \tag{B.7}$$

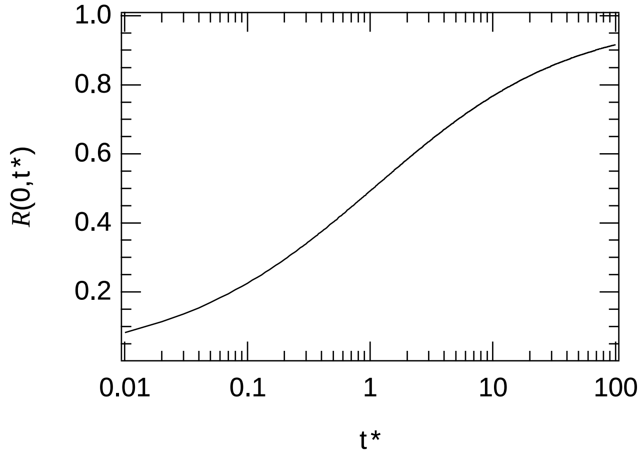
Within the effective size of the boundary layer, x_{eff} , where $y < \sqrt{\pi}/4$ (see eq. (2.39)), we see in fig. B.1 that for a given t R practically remains constant.

Figure B.1 R within x_{eff}.



In fig. B.2 the behaviour of R at $x = 0$ is displayed as a function of t^*. From fig. B.1 we may conclude that this behaviour is characteristic for the boundary layer.

Figure B.2 R at x=0.



Now that we have explored the second ratio in (B.1), we turn our attention to the first. We may consider two limiting cases. For $\sigma_i >> 1$, or far from the critical point, this ratio reduces to a ratio between the surface of the heater S_h and the sum of surfaces of all surrounding walls S_{tot}. For $\sigma_i << 1$, or close to the critical point, this ratio yields

$$\frac{\sigma_h S_h}{\sum_{i=1}^{N} \sigma_i S_i} = \frac{\frac{\lambda_h}{\sqrt{D_h}} S_h}{\sum_{i=1}^{N} \frac{\lambda_i}{\sqrt{D_i}} S_i}. \tag{B.8}$$

The thermal impedance and the surface of the heater substrate are readily available. As adopted in section 6.2.3, instead of trying to find for each wall segment the thermal impedance and the surface, the net effect of all walls is described by a single set of phenomenological parameters referred to as apparent values. Using the values given in table 6.1, for the two limiting cases, we have:

$$\frac{S_h}{S_a} \approx 8\% \quad \text{for } \sigma_i >> 1 \tag{B.9}$$

$$\frac{\lambda_h}{\lambda_a}\sqrt{\frac{D_a}{D_h}}\frac{S_h}{S_a} \approx 8‰ \quad \text{for } \sigma_i << 1 \,. \tag{B.10}$$

We may conclude that the combined value for the term described by (B.1), within the effective size of the boundary layer and for times $t < 10t_c$, is always smaller than 5%, decreasing on approaching the critical point. Moreover, far from the critical point σ_h is of order unity so that, in that region, (B.1) is even a factor of two smaller, i.e. 2.5%.